

REC'D OCT 1 1930

LANGLEY CUB-LIBRARY

14-349
NACA 349

TECHNICAL NOTES
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

CASE FILE
COPY

No. 349

AN INVESTIGATION OF AIRPLANE LANDING SPEEDS

By Kenneth F. Ridley

NACA LIBRARY
LANGLEY AERONAUTICAL LABORATORY

Washington
September, 1930

TN-349

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

TECHNICAL NOTE NO. 349.

AN INVESTIGATION OF AIRPLANE LANDING SPEEDS.

By Kenneth F. Ridley.

S u m m a r y

This paper describes an investigation on airplane landing speeds which was made to determine the applicability of accepted aerodynamic theory to the prediction of this particular performance characteristic.

The experimental work consisted in measuring the landing speed of several monoplanes by a new photographic method. The results of these tests supplemented by available information regarding biplanes were compared with predictions made in accordance with basic aerodynamic theory. The prediction makes use of the fundamental relation between wing loading, lift coefficient, and speed of level flight, and the effects of aspect ratio and proximity to the ground on lift curve slope.

Very satisfactory agreement was found between the predicted and measured values.

A set of generalized lift curves developed to simplify the method of prediction are presented in an appendix.

*Thesis submitted in partial fulfillment of the requirements for the degree of Engineer in Mechanical Engineering Aeronautics, Stanford University.

I n t r o d u c t i o n

Probably less is known and more is erroneously stated about the landing speed than is the case for any other airplane performance characteristic. The prevalence of obviously erroneous and sometimes even absurd specifications has been pointed out in References 1 and 2. The state of knowledge is well indicated by the somewhat common practice of predicting landing speeds on the basis of maximum lift coefficients determined by model airfoil tests made at low Reynolds Number, no account being taken of the angle of attack in the landing attitude. It appears that this situation may be largely attributed to the lack of a simple method for obtaining experimental information on the subject.

The only known source of accurate information on airplane landing speeds is given in Reference 4. Tests of a number of military biplanes are described in this report but no analysis of the results is included.

The program of the present investigation was to supplement the available data with the results of tests on various types of monoplanes, to make predictions of landing speeds for all the tested airplanes and to compare the predicted and measured values.

Experimental Methods and Apparatus

The biplane landing speeds reported by Carroll were determined by the use of synchronized recording flight instruments, namely, an air-speed meter, an accelerometer to indicate the instant of impact, and a ground-speed meter used as an air-speed check and to furnish wind-speed data. It is not definitely stated in the report whether a single representative landing or an average of several tests furnished the final results. The degree of accuracy is stated as plus or minus 3 per cent.

The method used by the writer was to determine the ground speed from a photographic record of the rotation of the landing wheel, by the use of a motion-picture camera which operated at a known constant speed, to measure the wind speed and to fix the still air landing speed as the algebraic sum of these quantities.

The procedure was to paint the wheel or tire of the airplane in contrasting colors, thus providing a rotation indicator, and then to photograph the airplane while making normal landings; wind speeds were simultaneously read from an anemometer. The duration of filming was from a second or two before to four or five seconds after impact.

The film was examined frame by frame, the position of the wheel in each picture recorded and a curve of wheel revolutions vs. exposures was plotted from these data. Knowledge of the

camera speed and tire circumference makes this curve in effect, a space-time curve from which velocities may be derived by graphical differentiation. It was found that the wheel acquired within one revolution a speed which varied negligibly during the next second or two. The slope of this portion of the curve was therefore taken to represent the speed at contact.

The motion-picture camera used was a portable, spring-driven type which operated at approximately 34 frames per second. A special electric clock was used to calibrate the camera. The exposure rate was determined for all conditions under which tests were likely to be made, i.e., from end to end of a full loading of film, and check calibrations were made at the beginning and end of each spool of film or each set of tests.

A vane, or windmill type of anemometer was used to measure the ground wind speed. Since this type of instrument registers only the revolutions made by the vanes, the wind speeds actually are average velocities over a period of about 10 seconds which includes the instant of landing.

Method of Prediction

The method of prediction is based on the following conception of a perfectly executed three-point landing. The airplane approaches the landing area in a steady glide. On nearing the ground the flight path becomes practically horizontal, the angle of attack increasing and the speed decreasing in such a way as

to maintain constant lift until the attitude for the three-point landing (wheels and tail skid at the same height) is reached. This attitude is maintained while the speed and lift decrease sufficiently to let the airplane settle to the ground. The landing speed is therefore assumed to be the speed of level flight at a very small height above the ground at the angle of attack corresponding to a three-point landing.

It is known that upon approaching the ground there is a reduction of the induced angle of attack of the wings of an airplane which may be calculated according to the multiplane theory of Prandtl. This produces an increase of lift curve slope and a reduction of induced drag; it is therefore equivalent to an increase in the aspect ratio.

The calculation of this effective aspect ratio which is representative of level flight just above the ground is the first step of the prediction. It is next necessary to develop a curve of lift coefficient vs. angle of attack for a wing of this aspect ratio. The lift coefficient which the wings may be expected to attain is the value indicated by the curve at the angle of attack corresponding to a three-point landing.

It is now necessary to make some assumption regarding the relation of wing lift to the total lift of the airplane for the landing condition. It does not appear reasonable to assume that the two are equal because the sum of the vertical air loads on the fuselage and horizontal tail surfaces is usually too

large to be neglected. Since the air forces on the tail surfaces at high angles of attack and with conventional center of gravity location are small, and since the lift of a normal type of fuselage is considerable at high angles of attack, it is reasonable to assume that the wings support somewhat less than the total weight of the airplane. The lift of the parasite parts might be approximated by crediting the airplane with the wing area it would have if the wing were not interrupted, by the fuselage. For the prediction of landing speeds, then, it is assumed that the effective wing area is that of the complete plan form of the wings, i.e., no deduction will be made for area blanketed by the fuselage.

The required data, namely, weight, wing area, and lift coefficient now being known, the landing speed is predicted according to the equation

$$V = \sqrt{\frac{2 W}{\rho C_L S}}$$

wherein

V = velocity (feet per second)

W = weight of airplane (pounds)

ρ = density of air (slugs per cubic foot)

C_L = lift coefficient

S = wing area (square feet).

The effective aspect ratio corresponding to level flight just above the ground is calculated as follows. The reduction

of the induced angle of attack of a wing system upon approaching the ground is

$$\Delta \alpha_i = -\bar{\sigma} \frac{C_L S}{\pi b_1^2}$$

wherein

$\Delta \alpha_i$ = change of induced angle of attack (radians)

b_1 = maximum span (feet)

$\bar{\sigma}$ = influence coefficient.

For the monoplane, $\bar{\sigma}$ is equal to σ , the coefficient of mutually induced drag.* For the biplane $\bar{\sigma}$ is given by the equation

$$\bar{\sigma} = \frac{\sigma_{1-1'} + \sigma_{2-2'} + 2\sigma_{1-2'}}{4}$$

The subscripts follow the convention used by Wieselsberger (Reference 3), that is, $\sigma_{1-1'}$ is the influence coefficient for a biplane composed of one of the wings of the biplane and its mirror image in the ground plane, $\sigma_{2-2'}$ corresponds to a similar biplane composed of the other wing and its image, and $\sigma_{1-2'}$ refers to a biplane composed of one of the real wings and the mirror image of the other.

Strictly speaking, the above formulas are applicable only to monoplanes and equal-span biplanes having ideal lift distribution. However, the expressions may be used in practice with no appreciable error for all normal biplanes and excepting only the extreme sesquiplane types.

From the equation for the induced angle of attack of a

*See Appendix II for values of σ .

monoplane

$$\alpha_1 = \frac{C_L S}{\pi b^2} \quad \left(\text{or} \quad \frac{C_L}{\pi R} \right)$$

it may be deduced that the equivalent aspect ratio when near the ground is

$$R_e = \frac{R}{1 - \sigma}$$

and an analogous derivation for the biplane gives

$$R_e = \frac{R}{1 - \sigma k^2}$$

R = aspect ratio of the real or equivalent monoplane.

R_e = effective aspect ratio of real or equivalent monoplane near the ground.

k = Munk's biplane constant.

The determination of the lift curve corresponding to this effective aspect ratio is accomplished by application of the well-known formula

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{R_e} - \frac{1}{R_1} \right) 57.3$$

to the results of full scale (Variable Density Wind Tunnel) tests of the airfoil profile used in the airplane. In this formula

α_2 = angle of attack of airplane wing (degrees).

α_1 = angle of attack of airfoil model tested (degrees).

R_1 = aspect ratio of model tested (after correction for tunnel wall effect).

R_e = effective aspect ratio of airplane wing (or that of equivalent monoplane).

R e s u l t s

The results of the investigation are presented in full in Appendix I; a condensed summary will be found in the next section.

The results of the experiments and calculations, together with the important design characteristics of the airplanes, are given in Tables II and III. The computations of the predicted landing speeds occupy pages 17-37, while the predicted lift curves are shown in Figures 1-11.

Examples of the film records appear as Figures 16-19.

Typical curves of wheel revolutions vs. exposures as derived from the pictures are presented as Figures 12-15.

The monoplane results were derived from only such landings as were found by examination of the films to be normal three-point landings. The average of at least two such landings was used as the final result for each airplane. The landing angles, wing areas, and weights were measured at the time of making the tests, but in some cases the angle of zero lift was obtained from the manufacturer.

D i s c u s s i o n

For convenience of reference a summary of the results is presented in Table I.

TABLE I

Airplane	C_L (actual)	C_L (predicted)	Actual landing speed (m.p.h.)	Predicted landing speed (m.p.h.)
Lockheed (Sirius)	1.40	1.373	57.3 57.0 Av. (57.15)	57.8
Ford (4 AT-E)	1.39	1.33	59.1 58.4 Av. (58.75)	60.1
Hamilton (H-45)	1.282	1.260	54.8 61.2 59.5 Av. (58.5)	59.0
Stinson (Jr.)	1.462	1.35	54.7 53.9 56.1 Av. (54.9)	57.1
SE-5A	1.10	1.15	54.0	52.9
JN-6H	1.178	1.18	51.0	51.0
VE-7	1.10	1.142	51.0	50.1
DH-4b	1.095	1.123	56.5	55.9
Sperry Messenger	1.22	1.163	44.0	45.1
MB-3	1.142	1.182	57.0	56.0
MB-2	1.098	1.195	58.0	55.6

The results of tests of two additional airplanes - the Fokker CO-4 and the S.P.A.D. VII - appear in Carroll's report; inability to obtain complete design characteristics prevented the prediction of their landing speeds. This is particularly unfortunate in the case of the S.P.A.D. since its landing lift coefficient is unusually small ($C_L = 0.98$). This value, however, would appear to be consistent with the use of a very thin airfoil of small camber in a biplane of low equivalent aspect ratio which has only a medium value of the landing angle.

On examination of the above table, it will be noted that there is a maximum variation predicted and experimental values of less than $2\frac{1}{2}$ miles per hour in landing speed and 10 per cent in lift coefficient. It would appear unreasonable to expect better agreement than this in view of the probable accuracy of the test data.

The experimental accuracy of the biplane results is stated as plus or minus 3 per cent. The principal sources of error in the results obtained by the writer are probably variation of the camera speed and of wind velocity during a test. Different calibrations of the camera showed noticeable differences in speed between different spools although no important variation was detected within any one. It will be remembered that the average over a short period, including the instant of contact, was used as the instantaneous wind velocity. If the wind were not steady, the instantaneous velocity might vary consider-

ably from this average. One other possibility for error is seen in the use of the no-load tire circumference in the calculations of the ground speed. It is necessarily small, however, because at the speed and attitude of landing, the load on the wheels can only be a very small fraction of the total weight of the airplane. In most cases the full load radial deflection of the tire corresponds to a reduction of circumference of not more than 4 or 5 per cent.

Errors in the predictions themselves may result from several causes, the principal one being the assumption of a fictitious wing area to compensate for the lift produced by the parasite parts. Other possible sources of error are failure to reproduce in the airplane the profile of the specified airfoil, inaccuracies in the model test data, and the uncertainty of assumed airfoil characteristics when no full scale data was available for the profile used. In consideration of the possible sources of inaccuracy in both the predicted and actual values, the method of prediction is considered highly satisfactory.

With the validation of the theoretical principles utilized for prediction, it becomes possible to enumerate and discuss the importance of the design characteristics which control airplane landing speeds. These variables are wing loading, airfoil profile characteristics, landing angle of attack, aspect ratio and wing height.

In reality there are only two entirely distinct variables,

wing loading and lift coefficient, as indicated by the fundamental equation

$$V = \sqrt{\frac{2}{\rho} \left(\frac{W}{S}\right) \left(\frac{1}{C_L}\right)}.$$

No discussion of the first seems necessary. The lift coefficient attainable in landing is fixed by the characteristics of the wing profile, the equivalent aspect ratio when close to the ground, and the angle of attack in the three-point landing attitude. The possibility of reducing landing speeds by the use of slotted or other forms of high lift wings is clearly indicated, and the necessity for consideration of the effective aspect ratio in fixing the landing angle is likewise apparent. It will be noticed that the theory indicates no effect of wing height upon the landing speed which could not be obtained otherwise, that is, by adjustment of the landing angle. The small influence of wing height upon lift curve slope for the aspect ratios common to existing monoplanes may be seen in the predictions for these airplanes.

C o n c l u s i o n

The landing speed of an airplane can be predicted with satisfactory accuracy by the use of accepted aerodynamic theory.

Note.— Generalized curves to facilitate the prediction of landing speeds are given in Appendix II.

References and Bibliography

- Reference 1. Reid, E. G. : An Analysis of Airplane Landing Speeds. Aviation, July 20, 1929.
- Reference 2. Lee, F. G. : Landing Speed of Airplanes. Transactions of American Society of Mechanical Engineers, April-June, 1929, Vol. I, No. 2.
- Reference 3. Wieselsberger, C. : Wing Resistance Near the Ground. N.A.C.A. Technical Memorandum No. 77, 1922.
- Reference 4. Carroll, Thomas : A Comparison of the Take-Off and Landing Characteristics of a Number of Service Airplanes. N.A.C.A. Technical Report No. 249, 1927.
- Prandtl, L. : Applications of Modern Hydrodynamics to Aeronautics. N.A.C.A. Technical Report No. 116, 1921.
- Raymond, A. E. : Ground Influence on Aerofoils. N.A.C.A. Technical Note No. 67, 1921.
- Betz, A. : The Lift and Drag of a Wing in Proximity to the Ground. McCook Field Memorandum Report No. 167, 1925.

Appendix I

TABLE II. Collected Results (Monoplane)

Airplane	Gross weight lb.	Wing area sq.ft.	Wing loading lb./sq.ft.	Ground speed m.p.h.	Wind speed m.p.h.	Actual landing speed m.p.h.	Predicted landing speed m.p.h.	C_L (actual)	C_L (predicted)
Lockheed (Sirius)	3415	288	11.86	59.1 59.0	-1.8 -2.0	57.3 57.0 Av.(57.15)	57.8	1.40	1.373
Ford (4AT-E)	10085	8.18	12.32	59.1 58.4	0 0	59.1 58.4 Av.(58.75)	60.1	1.39	1.33
Hamilton (H-45)	4536	405	11.21	54.8 57.7 58.4	0 3.5 1.1	54.8 61.2 59.5 Av.(58.5)	59.0	1.282	1.260
Stinson (Jr.)	2797	248	11.29	54.7 52.4 45.8	0 1.5 10.5	54.7 53.9 56.1 Av.(54.9)	57.1	1.462	1.35

TABLE III. Collected Results (Biplane)

Airplane	Gross weight lb.	Wing area sq.ft.	Wing loading lb./sq.ft.	Actual landing speed m.p.h.	Predicted landing speed m.p.h.	C_L (actual)	C_L (predicted)
SE-5A	2080	254	8.19	54	52.9	1.10	1.15
JN-6H	2767	354	7.83	51	51.0	1.178	1.18
VE-7	2152	295	7.29	51	50.1	1.10	1.142
DH-4b	4000	450	8.89	56.5	55.9	1.095	1.123
Sperry Messenger	965	160	6.03	44	45.1	1.22	1.163
MB-3	2277	242	9.43	57	56.0	1.142	1.182
MB-2	10520	1121	9.39	58	55.6	1.098	1.195

Ford 4-A-T Prediction

Specifications

High-wing cabin monoplane (trimotored).

Wing profile	U.S.A. 27B at root Göttingen 387 at tip*
Span	74 ft.
Area	818 sq.ft.
Height of wing	9.5 ft.
Weight as flown	10,085 lb.
Wing loading	11.21 lb./sq.ft.
Ground angle	18 deg. from O lift.

Ground Effect

$$\mu = 1$$

$$\frac{h_{1-1}'}{b} = \frac{2 \times 9.5}{74} = 257$$

$$\sigma = .416$$

$$\begin{aligned} Re &= \frac{R}{1 - \sigma} = \frac{b^2}{S} \times \frac{1}{1 - \sigma} \\ &= \frac{74^2}{818} \times \frac{1}{1 - .416} = 11.48 \end{aligned}$$

Lift Curve Correction

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{Re} - \frac{1}{R_1} \right) 57.3$$

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{11.48} - \frac{1}{6.85} \right) 57.3$$

$$= \alpha_1 - 1.075 C_L$$

*Göttingen 387 characteristics used.

α_1 (deg.)	C_L	$-1.075C_L$	α_2 (deg.)
-6.8	0	- 0	-6.8
-3	.28	- .301	-3.3
0	.50	- .538	-0.538
3	.723	- .776	2.224
6	.951	-1.021	4.979
9	1.153	-1.236	7.764
12	1.305	-1.403	10.597
15	1.330	-1.430	13.570
18	1.318	-1.417	16.583

Hamilton H-45 Prediction

Specifications

High-wing cabin monoplane

Wing section	Clark Y modified*
Span	54.25 ft.
Area	405 sq.ft.
Height of wing	7.75 ft.
Weight as flown	4536 lb.
Wing loading	11.21 lb./sq.ft.
Ground angle	11 degrees

*Clark Y characteristics used.

Ground Effect

$$\frac{h_{1-1'}}{b} = \frac{2 \times 7.75}{54.25} = .286 \quad \mu = 1 \quad \sigma = .388$$

$$Re = \frac{R}{1 - \sigma} = \frac{b^2}{S} \times \frac{1}{1 - \sigma}$$

$$= \frac{54.25}{405} \times \frac{1}{1 - .388} = 11.88$$

Lift Curve

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{Re} - \frac{1}{R_1} \right) 57.3$$

$$= \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{11.88} - \frac{1}{6.85} \right) 57.3 = \alpha_1 - 1.127 C_L$$

α_1 (deg.)	C_L	$-1.127 C_L$	α_2 (deg.)
-5.2	0	-0	-5.2
-3.0	.162	-.182	-3.182
0	.380	-.428	-0.428
3	.600	-.675	2.325
6	.820	-.924	5.076
9	1.033	-1.065	7.935
12	1.230	-1.386	10.614
15	1.366	-1.538	13.462
18	1.284	-1.448	16.552

Lockheed Sirius Prediction

Specifications

Low-wing open monoplane

Wing profile	tapered Clark Y*
Area	288 sq.ft.
Span	41.9 ft.
Height of wing	4 ft.
Weight as flown	3415 lb.
Wing loading	11.86 lb./sq.ft.
Ground angle	15 degrees

Ground Effect

$$\frac{h_{1-1'}}{b} = \frac{8}{41.9} = .191 \quad \mu = 1 \quad \sigma = .5$$

$$Re = \frac{b^2}{S} \times \frac{1}{1 - \sigma} = \frac{41.9}{288} \times \frac{1}{.5} = 12.18$$

Lift Curve

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{Re} - \frac{1}{R_1} \right) 57.3$$

$$= \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{12.18} - \frac{1}{6.85} \right) 57.3 = \alpha_1 - 1.162 C_L$$

*Clark Y characteristics used.

α_1 (deg.)	C_L	$-1.162C_L$	α_2 (deg.)
-5.2	0	0	-5.2
-3.0	.162	-.188	-3.188
0	.380	-.442	-.442
3	.600	-.699	2.301
6	.820	-.955	5.045
9	1.033	-1.205	7.795
12	1.230	-1.431	10.569
15	1.366	-1.588	13.412
18	1.284	-1.495	16.505

Stinson Jr. Prediction

Specifications

Cabin monoplane

Wing profile	Clark Y
Span	41.8 ft.
Area	248 sq.ft.
Height of wing	7.5 ft.
Weight as flown	2797 lb.
Wing loading	11.29 lb./sq.ft.
Ground angle	13.1 degrees

Ground Effect

$$\frac{h_1 - i'}{b} = \frac{15}{41.7} = .359$$

$$\mu = 1 \quad \sigma = .320$$

$$\begin{aligned} R_e &= \frac{R}{1 - \sigma} = \frac{b^2}{S} \times \frac{1}{1 - \sigma} \\ &= \frac{41.7}{248} \times \frac{1}{1 - .320} = 10.3 \end{aligned}$$

Lift Curve

$$\begin{aligned} \alpha_2 &= \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{R_e} - \frac{1}{R_1} \right) 57.3 \\ &= \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{10.3} - \frac{1}{6.85} \right) 57.3 \\ &= \alpha_1 - .875 C_L \end{aligned}$$

α_1 (deg.)	C_L	$-.875C_L$	α_2 (deg.)
-5.2	0	0	-5.2
-3.0	.162	-.142	-3.142
0	.380	-.323	-0.323
3	.600	-.525	2.475
6	.820	-.718	5.282
9	1.033	-.905	8.095
12	1.230	-1.076	10.924
15	1.366	-1.196	13.804
18	1.284	-1.125	16.875

Sperry Messenger Prediction

Specifications

Open biplane (light one-seater)

Wing profile	U.S.A.-5
Area	160 sq.ft.
Span (both wings)	20 ft.
Gap	2.83 ft.
Chord (both wings)	4.0 "
Height of lower wing	2.1 "
Weight as flown	965 lb.
Wing loading	6.03 lb./sq.ft.
Ground angle	17.2 deg.

Biplane Constant

$$\frac{2h}{b_1 + b_2} = \frac{2 \times 3.83}{2 \times 26} = .1915$$

$$\mu = \frac{b_1}{b_2} = 1$$

$$\sigma = .498$$

Since it is of equal span and equal chord

$$\begin{aligned} \frac{1}{k^2} &= \frac{1}{2} (1 + \sigma) \\ &= \frac{1.498}{2} = .749 \end{aligned}$$

$$k^2 = 1.335$$

Ground Effect

Wings	h	$\frac{2h}{b_1 + b_2}$	μ	σ
1 - 1'	11.86	.593	1	.187
2 - 2'	4.2	.21	1	.471
1 - 2'	8.03	.401	1	.288

$$\bar{\sigma} = \frac{\sigma_{1-1'} + \sigma_{2-2'} + 2 \sigma_{1-2'}}{4}$$

$$= \frac{.187 + .471 + 2 \times .288}{4} = .3085$$

$$Re = \frac{k^2 b^2}{s} \times \frac{1}{1 - \bar{\sigma} k^2}$$

$$= \frac{1.335 \times 20}{160 (1 - .3085 \times 1.335)} = 5.68$$

Lift Curve

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{Re} - \frac{1}{R_1} \right) 57.3$$

$$= \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{5.68} - \frac{1}{6.85} \right) 57.3 = \alpha_1 + .547 C_L$$

α_1 (deg.)	C_L	$.547C_L$	α_2 (deg.)
-4.2	0	0	-4.2
0	.325	.177	0.177
3	.555	.304	3.304
6	.780	.426	6.426
9	1.000	.547	9.547
12	1.190	.651	12.651
15	1.185	.649	15.649
18	1.135	.621	18.621

JN - 6H Prediction

Specifications

Open biplane (primary training)

Wing profile	Eiffel - 36*
Span, upper	43.7 ft.
" lower	32.7 "
Gap	5.17 "
Height of lower wing	2.35 "
Wing area	354 sq.ft.
Wing loading	7.83 lb./sq.ft.
Ground angle of attack	13.2 deg.

*U.S.A.-5 characteristics used.

Equivalent Monoplane

$$\frac{2 h}{b_1 + b_2} = \frac{2 \times 5.17}{43.7 + 32.7} = .1355$$

$$\mu = \frac{b_2}{b_1} = \frac{32.7}{43.7} = .748$$

$$\sigma = .52$$

$$\frac{1}{k^2} = \frac{1 - \sigma^2}{1 - 2 \sigma \mu + \mu^2} = \frac{1 - .52^2}{1 - 2 \times .52 \times .748 + .748^2}$$

$$= \frac{.73}{.781} = .935$$

$$k^2 = 1.07$$

Ground Effect

Wings	h	$\frac{2 h}{b_1 + b_2}$	μ	σ
1 - 1'	15.04	.344	1.0	.330
2 - 2'	4.70	.1438	.748	.518
1 - 2'	9.87	.259	1.0	.412

$$\begin{aligned} \bar{\sigma} &= \frac{\sigma_{1-1'} + \sigma_{2-2'} + 2 \sigma_{1-2'}}{4} \\ &= \frac{.330 + .518 + 2 \times .412}{4} = .418 \end{aligned}$$

$$\begin{aligned} R_e &= \frac{k^2 b^2}{s (1 - \bar{\sigma} k^2)} \\ &= \frac{1.07 \times 43.7}{354 (1 - .418 \times 1.07)} = 10.47 \end{aligned}$$

Lift Curve

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{R_e} - \frac{1}{R_1} \right) 57.3$$

$$= \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{10.47} - \frac{1}{6.85} \right) 57.3 = \alpha_1 - .922C_L$$

α_1 (deg.)	C_L	$-.922C_L$	α_2 (deg.)
-2.2	0	0	-2.2
0	.163	-.150	-0.150
3	.392	-.362	2.638
6	.622	-.575	5.425
9	.851	-.785	8.215
12	1.063	-.983	11.017
15	1.210	-.118	13.882
18	1.01	-.933	17.067

SE - 5A Prediction

Specifications

Open biplane (pursuit)

Weight	2080 lb.
Area	254 sq.ft.
Wing loading	8.19 lb./sq.ft.
Gap	4.38 ft.
Span (both wings)	2.66 "
Height of lower wing	3.33 "
Landing angle	14 deg.
Wing profile	R.A.F. 15

Biplane Constant

$$\frac{2 h}{b_1 + b_2} = \frac{4.38}{26.6} = .1648$$

$$\mu = \frac{b_2}{b_1} = 1$$

$$\sigma = .538$$

$$\begin{aligned} \frac{1}{k^2} &= \frac{1}{2} (1 + \sigma) \\ &= \frac{1.538}{2} = .769 \end{aligned}$$

$$k^2 = 1.30$$

Ground Effect

Wings	h	$\frac{2 h}{b_1 + b_2}$	μ	σ
1 - 1'	17.42	.580	1	.192
2 - 2'	6.67	.250	1	.422
1 - 2'	11.04	.415	1	.279

$$\begin{aligned} \bar{\sigma} &= \frac{\sigma_{1-1'} + \sigma_{2-2'} + 2 \sigma_{1-2'}}{4} \\ &= \frac{.192 + .422 + 2 \times .279}{4} = .298 \end{aligned}$$

$$\begin{aligned} Re &= \frac{k^2 b^2}{S (1 - \bar{\sigma} k^2)} \\ &= \frac{130 \times 26.6}{254 (1 - .298 \times 1.30)} = 5.93 \end{aligned}$$

Lift Curve

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{R_e} - \frac{1}{R_1} \right) 57.3$$

$$= \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{5.93} - \frac{1}{6.85} \right) 57.3 = \alpha_1 + .373C_L$$

α_1 (deg.)	C_L	$373C_L$	α_2 (deg.)
-2.2	0	0	-2.2
0	.163	.061	0.061
3	.392	.146	3.146
6	.622	.232	6.232
9	.851	.318	9.318
12	1.063	.397	12.397
15	1.210	.451	15.451
18	1.010	.377	18.377

MB - 3 Prediction

Specifications

Open biplane (pursuit)

Wing section	R.A.F.-15
Span, upper	26 ft.
" lower	24.5 ft.
Gap	4.5 "
Height of lower wing	3.0 "
Wing area	242 sq.ft.
Wing loading	9.83 lb./sq.ft.
Ground angle of wings	15 deg.

Biplane Constant

$$\frac{2 h}{b_1 + b_2} = \frac{2 \times 4.5}{26 + 24.5} = 1.78$$

$$\mu = \frac{b_2}{b_1} = \frac{24.5}{26} = .942$$

$$\sigma = .508$$

$$\frac{1}{k^2} = \frac{1 - \sigma^2}{1 - 2 \sigma \mu + \mu^2}$$

$$\frac{1}{k^2} = \frac{1 - .508^2}{1 - 2 \times .942 \times .508 + .942^2} = .800$$

$$k^2 = 1.25$$

Ground Effect

Wings	h	$\frac{2 h}{b_1 + b_2}$	μ	σ
1 - 1'	15	.577	1.0	.193
2 - 2'	6	.245	.942	.423
1 - 2'	10.5	.416	1.0	.279

$$\begin{aligned} \bar{\sigma} &= \frac{\sigma_{1-1'} + \sigma_{2-2'} + 2 \sigma_{1-2'}}{4} \\ &= \frac{.193 + .423 + 2 \times .279}{4} = .293 \end{aligned}$$

$$\begin{aligned} Re &= \frac{k^2 b^2}{8 (1 - \bar{\sigma} k^2)} \\ &= \frac{1.25 \times 26}{232 (1 - .298 \times 1.25)} = 5.51. \end{aligned}$$

Lift Curve

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{R_e} - \frac{1}{R_1} \right) 57.3$$

$$= \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{5.51} - \frac{1}{6.85} \right) 57.3 = \alpha_1 + 6.47 C_L$$

α_1 (deg.)	C_L	$.647 C_L$	α_2 (deg.)
-3.2	0	0	-3.2
0	.163	.106	0.106
3	.393	.254	3.254
6	.622	.403	6.403
9	.851	.551	9.551
12	1.063	.680	12.680
15	1.210	.783	15.783
18	1.010	.654	18.654

VE-7 Prediction

Specifications

Open biplane (advanced training)

Wing section	R.A.F.-15
Area	295 sq.ft.
Span (both wings)	34.1 ft.
Gap	4.67 ft.
Height of lower wing	3.41 "
Weight as flown	2152 lb.
Wing loading	7.29 lb./sq.ft.
Ground angle	12.7 deg.

Biplane Constant

$$\frac{2 h}{b_1 + b_2} = \frac{2 \times 4.67}{2 \times 34.1} = .137$$

$$\mu = \frac{b_2}{b_1} = 1$$

$$\sigma = .538$$

$$\frac{1}{k^2} = \frac{1}{2} (1 + \sigma) = .792$$

$$k^2 = 1.262$$

Ground Effect

Wings	h	$\frac{2 h}{b_1 + b_2}$	μ	σ
1 - 1'	16.14	.474	1	.243
2 - 2'	6.8	.1995	1	.485
1 - 2'	11.47	.336	1	.337

$$\begin{aligned} \bar{\sigma} &= \frac{\sigma_{1-1'} + \sigma_{2-2'} + 2 \sigma_{1-2'}}{4} \\ &= \frac{.243 + .485 + 2 \times .335}{4} = .3505 \end{aligned}$$

$$\begin{aligned} Re &= \frac{k^2 b^2}{S (1 - \bar{\sigma} k^2)} \\ &= \frac{1.262 \times 34.1}{295 (1 - .3505 \times 1.262)} = 8.96 \end{aligned}$$

Lift Curve

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{R_e} - \frac{1}{R_1} \right) 57.3$$

$$= \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{8.96} - \frac{1}{6.85} \right) 57.3 = \alpha_1 - .629C_L$$

α_1 (deg.)	C_L	$-.629C_L$	α_2 (deg.)
-2.2	0	0	-2.2
0	.163	-.102	-0.102
3	.392	-.247	2.753
6	.622	-.391	5.609
9	.851	-.535	8.465
12	1.063	-.680	11.320
15	1.210	-.755	14.245
18	1.010	-.635	17.365

D-Hb Prediction

Specifications

Open biplane (observation)

Wing section	R.A.F.-15
Area	450 sq.ft.
Span (both wings)	42.5 ft.
Gap	5.5 "
Height of lower wing	3.5 "
Weight as flown	4000 lb.
Wing loading	8.89 lb./sq.ft.
Ground angle	12.3 deg.

Biplane Constant

$$\frac{2 h}{b_1 + b_2} = \frac{2 \times 5.5}{2 \times 42.5} = .1295$$

$$\mu = \frac{b_2}{b_1} = 1$$

$$\sigma = .590$$

$$\frac{1}{k^2} = \frac{1}{2} (1 + \sigma)$$

$$= .799$$

$$k^2 = 1.251$$

Ground Effect

Wings	h	$\frac{2 h}{b_1 + b_2}$	μ	σ
1 - 1'	18	.424	1	.274
2 - 2'	7	.165	1	.537
1 - 2'	12.5	.294	1	.375

$$\bar{\sigma} = \frac{\sigma_{1-1'} + \sigma_{2-2'} + 2 \sigma_{1-2'}}{4}$$

$$= \frac{.274 + .537 + 2 \times .375}{4} = .390$$

$$Re = \frac{k^2 b^2}{S (1 - \bar{\sigma} k^2)}$$

$$= \frac{1.251 \times 42.5}{450 (1 - .390 \times 1.251)} = 9.76$$

Lift Curve

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{R_e} - \frac{1}{R_1} \right) 57.3$$

$$= \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{9.76} - \frac{1}{6.85} \right) 57.3 = \alpha_1 - .793C_L$$

α_1 (deg.)	C_L	$-.793C_L$	α_2 (deg.)
-2.2	0	0	-2.2
0	.163	-.129	-0.129
3	.392	-.311	2.689
6	.622	-.493	5.507
9	.851	-.675	8.325
12	1.063	-.844	11.156
15	1.210	-.960	14.040
18	1.010	-.800	17.20

Martin MB-2 Prediction

Specifications

Open biplane (bimotored bomber)

Wing section	Albatros*
Span (both wings)	74.16 ft.
Gap	8.5 "
Total wing area	1121 sq.ft.
Weight as flown	10520 lb.
Wing loading	9.39 lb./sq.ft.
Height of lower wing	4.5 ft.
Ground angle	13 deg.

*U.S.A.-5 characteristics used.

Biplane Constant

$$\frac{2 h}{b_1 + b_2} = \frac{2 \times 8.5}{2 \times 74.16} = .1147$$

$$\mu = \frac{b_2}{b_1} = 1$$

$$\sigma = .620$$

$$\frac{1}{k^2} = \frac{1}{2} (1 + \sigma)$$

$$= \frac{1.628}{2} = .814$$

$$k^2 = 1.229$$

Ground Effect

Wings	h	$\frac{2 h}{b_1 + b_2}$	μ	σ
1 - 1'	26.0	.350	1	.328
2 - 2'	9.0	.1212	1	.615
1 - 2'	17.5	.236	1	.441

$$\bar{\sigma} = \frac{\sigma_{1-1'} + \sigma_{2-2'} + 2 \sigma_{1-2'}}{4}$$

$$= \frac{.328 + .615 + 2 \times .441}{4} = .456$$

$$Re = \frac{k^2 b^2}{S (1 - \bar{\sigma} k^2)}$$

$$= \frac{1.229 \times 74.16}{1121 (1 - .456 \times 1.229)} = 10.75$$

Lift Curve

$$\alpha_2 = \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{R_e} - \frac{1}{R_1} \right) 57.3$$

$$= \alpha_1 + \frac{C_L}{\pi} \left(\frac{1}{10.75} - \frac{1}{6.05} \right) 57.3 = \alpha_1 - .969 C_L$$

α_1 (deg.)	C_L	$-.969 C_L$	α_2 (deg.)
-4.2	0	0	-4.2
0	.325	-.315	-0.315
3	.555	-.538	2.462
6	.780	-.755	5.245
9	1.00	-.969	8.031
12	1.190	-1.152	10.848
15	1.185	-1.150	13.850
18	1.135	-1.100	16.900

A p p e n d i x II

The following curves (Fig. 21) have been prepared to facilitate the prediction of landing speeds. They constitute a set of generalized lift curves for effective aspect ratios of 4, 8, and 12, and for airfoil sections whose maximum lift coefficients are 1.0, 1.2, and 1.4. To use these curves the effective aspect ratio, the probable value of the maximum lift coefficient, and the landing angle of attack (measured from the angle of zero lift) must be known. From these data and a lift coefficient obtained by interpolation between the curves very satisfactory results may be produced.

The appended table illustrates, by application of the curves to the prediction of landing speeds for the airplanes treated in this paper, the accuracy which may be expected.

TABLE IV. Collected Results from Use of Generalized Lift Curves

Airplane	Wing loading lb./sq.ft.	C_L at landing deg.	Actual landing speed m.p.h.	Predicted landing speed m.p.h.	C_L (actual)	C_L (predicted)	Re	$C_{L_{max}}$
Lockheed (Sirius)	11.86	20.2	57.15	51.8	1.40	1.37	12.18	1.39
Ford-4AT	12.32	18.0	58.75	60.3	1.39	1.32	11.48	1.33
Hamilton (H-45)	11.21	17.2	58.2	57.6	1.28	1.315	11.88	1.39
Stinson (Jr.)	11.29	18.3	54.9	57.1	1.46	1.35	10.3	1.39
SE-5A	8.19	16.2	54.0	53.5	1.10	1.12	5.93	1.21
JN-6H	7.83	15.4	51.0	51.5	1.178	1.16	10.47	1.21
VE-7	7.29	14.9	51.0	50.5	1.10	1.12	8.96	1.21
DH-4b	8.89	14.5	56.5	55.9	1.095	1.12	9.76	1.21
Sperry Messenger	6.03	21.4	44.0	45.5	1.22	1.14	5.68	1.19
MB-3	9.43	17.2	57.0	56.5	1.142	1.16	5.51	1.21
MB-2	9.39	17.2	58.0	55.5	1.098	1.20	10.75	1.19

June, 1930.

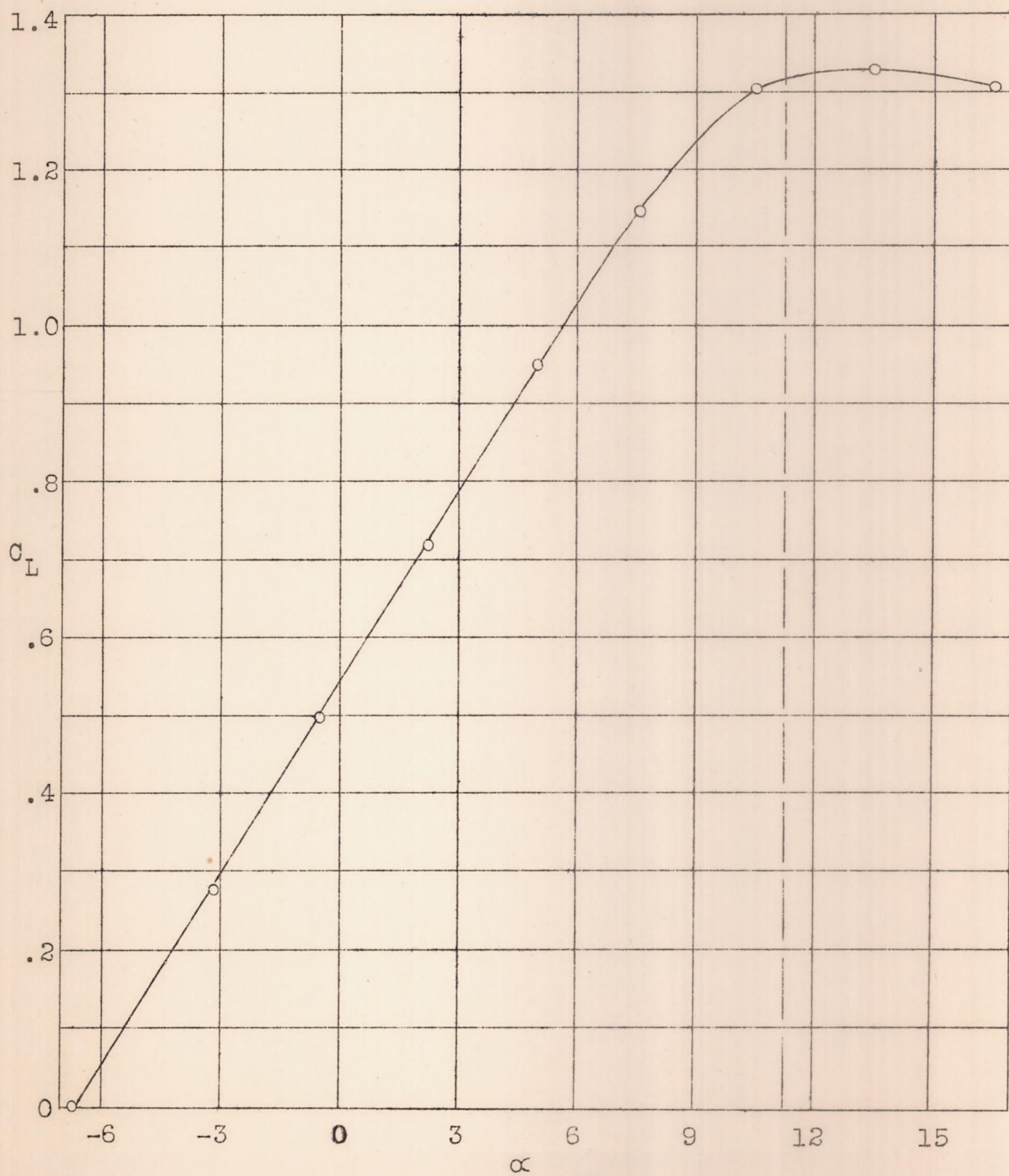


Fig. 1

Ford 4-AT

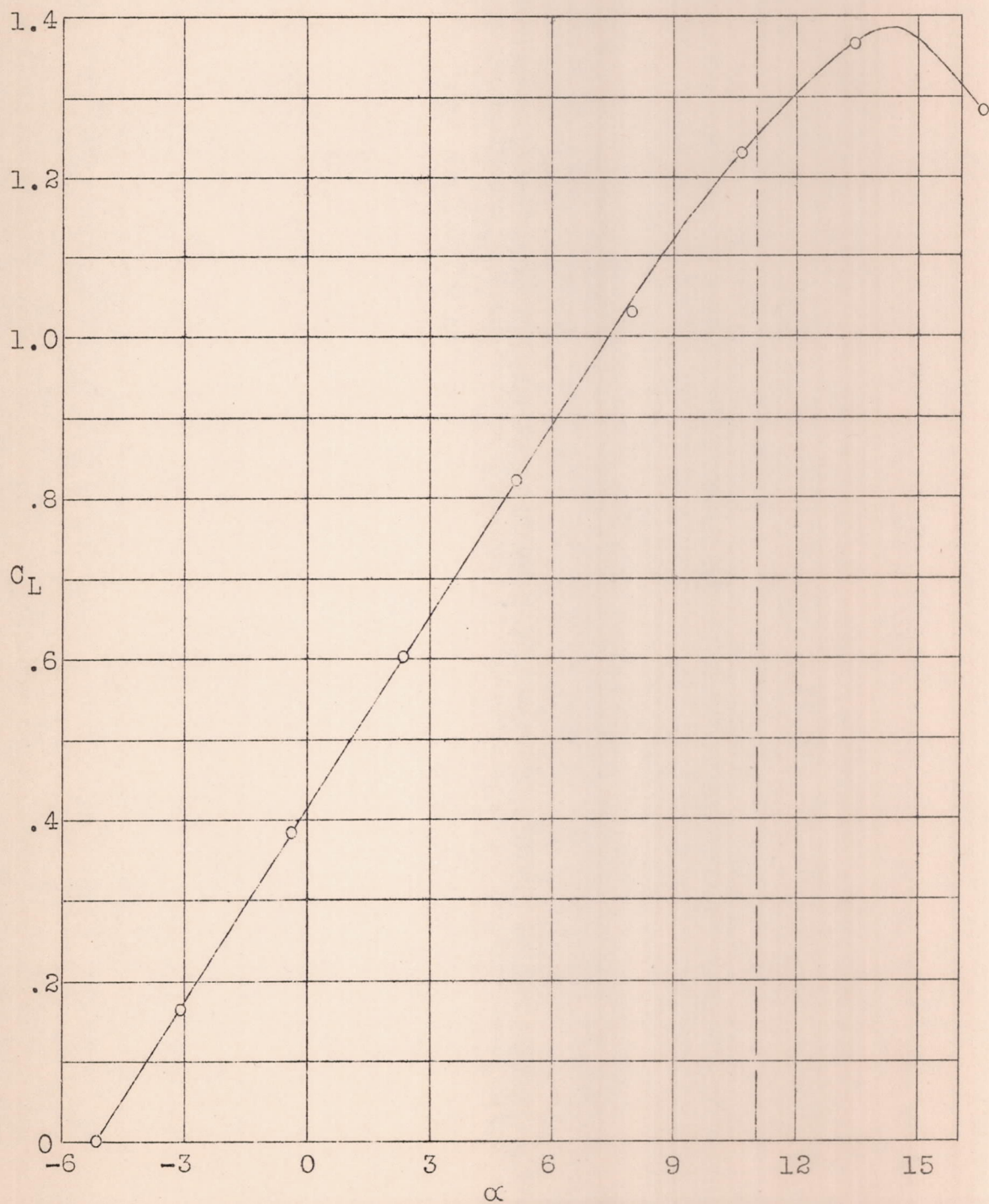


Fig. 2

Hamilton H-45

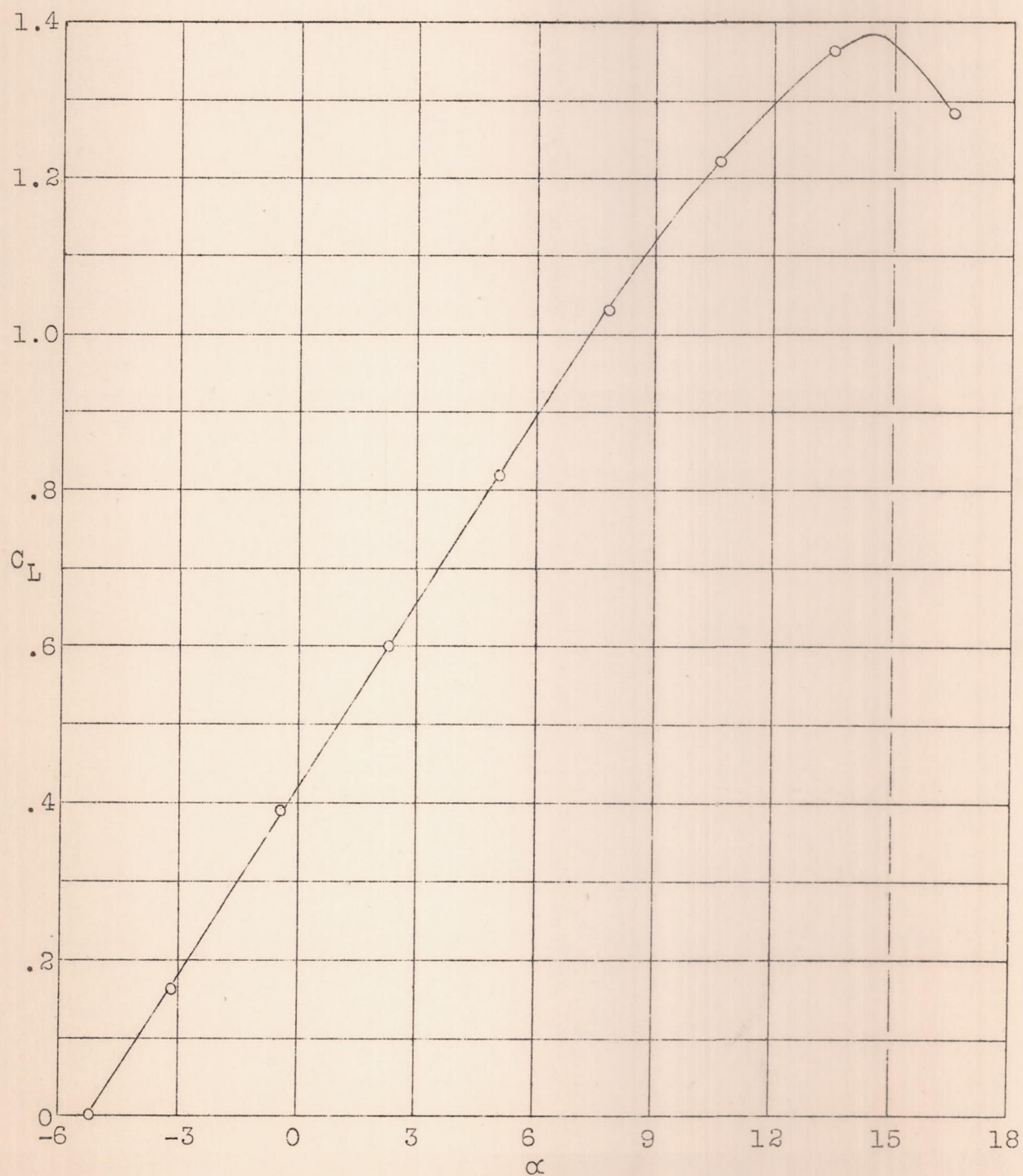


Fig. 3

Lockheed Sirius

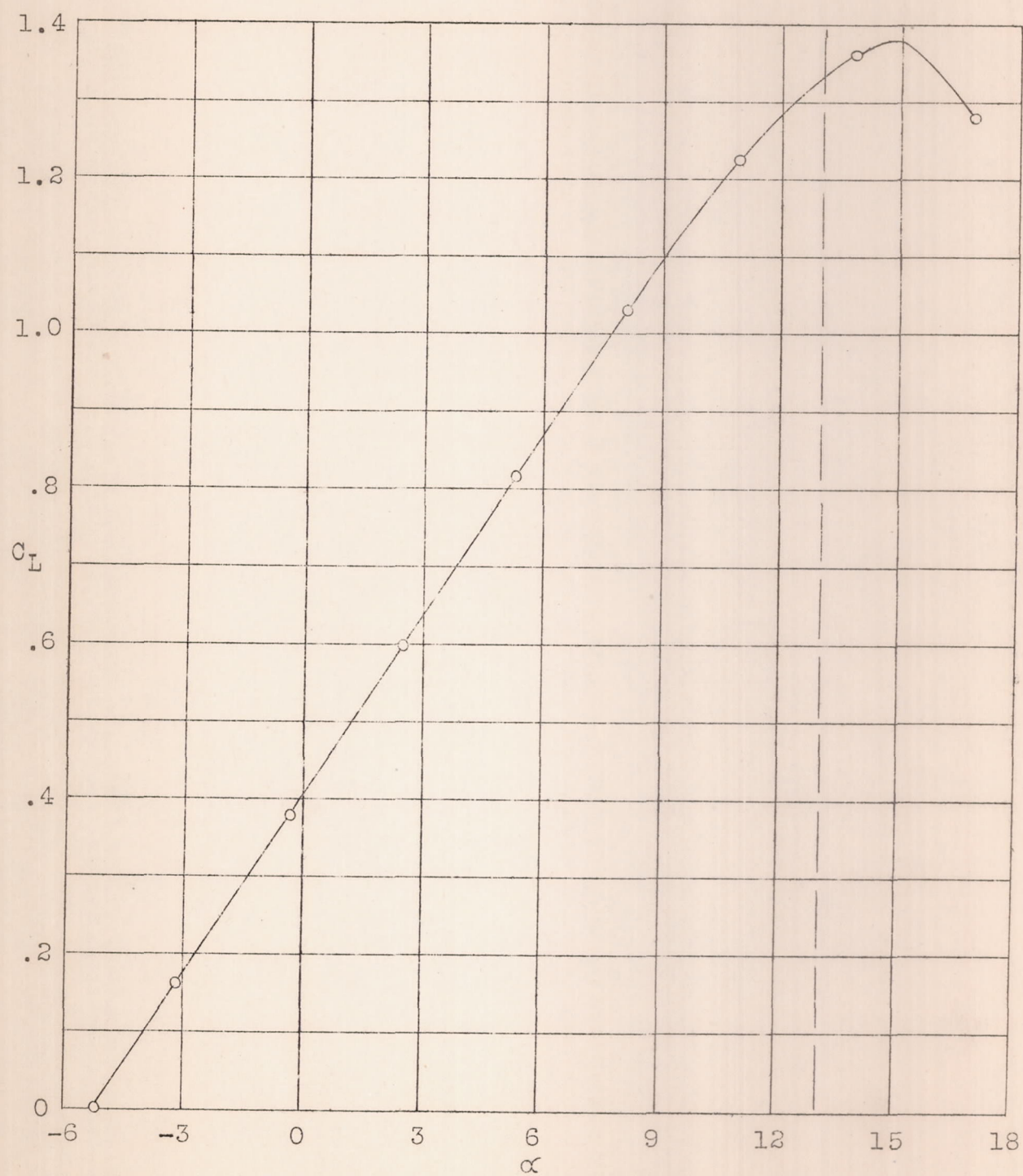


Fig. 4

Stinson Jr.

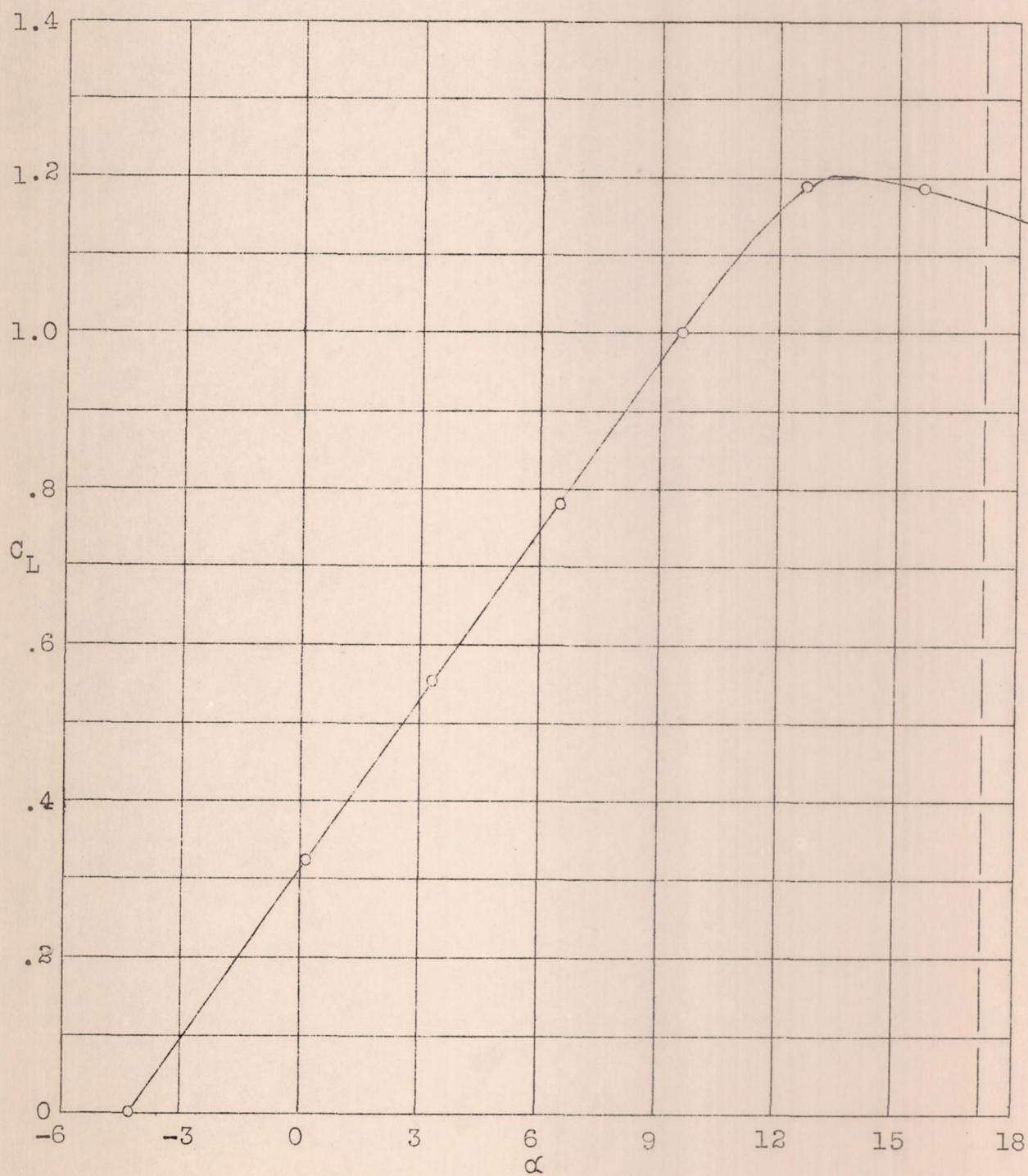


Fig. 5

Sperry Messenger

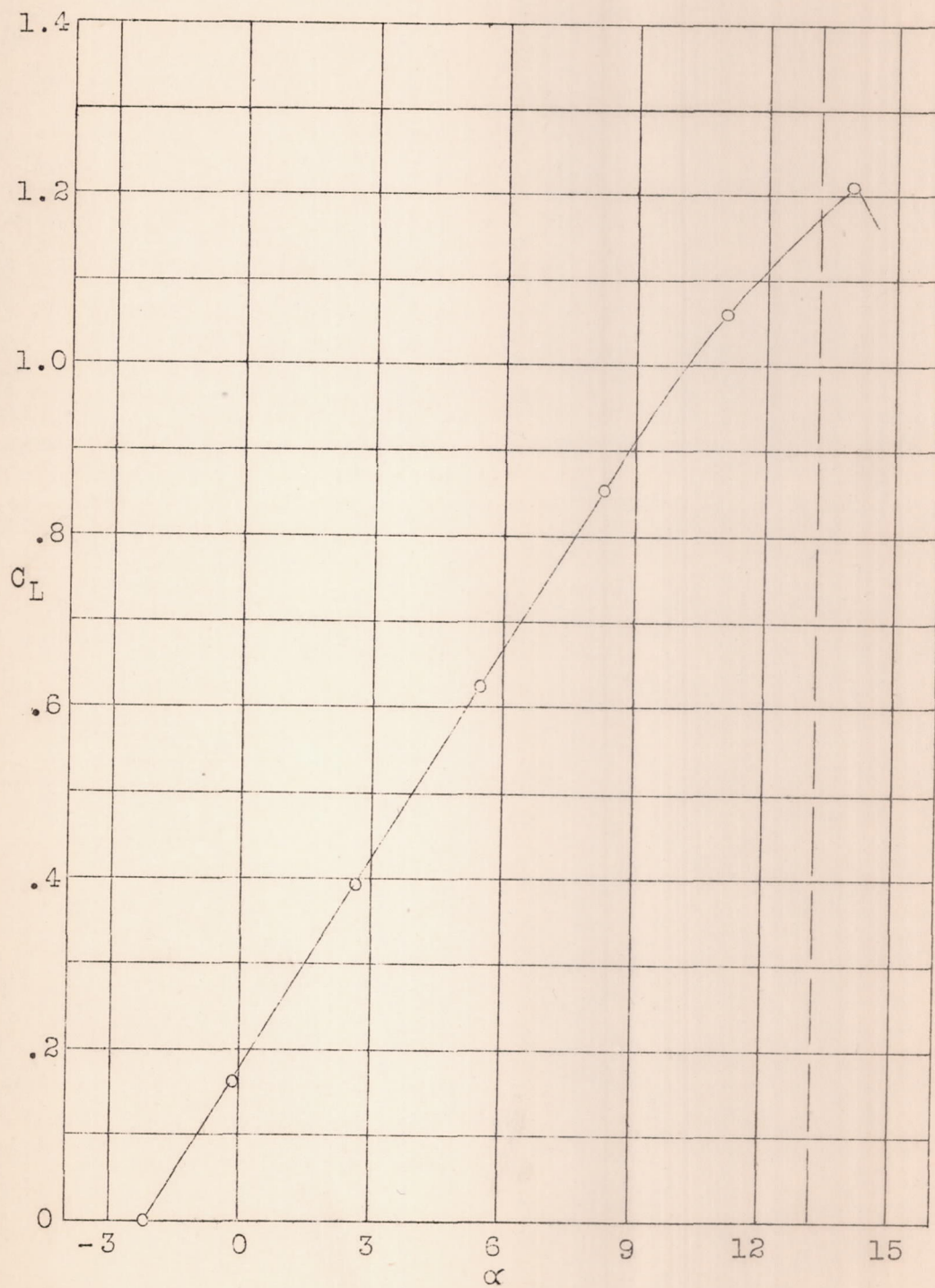


Fig. 6

Curtiss JW 6H

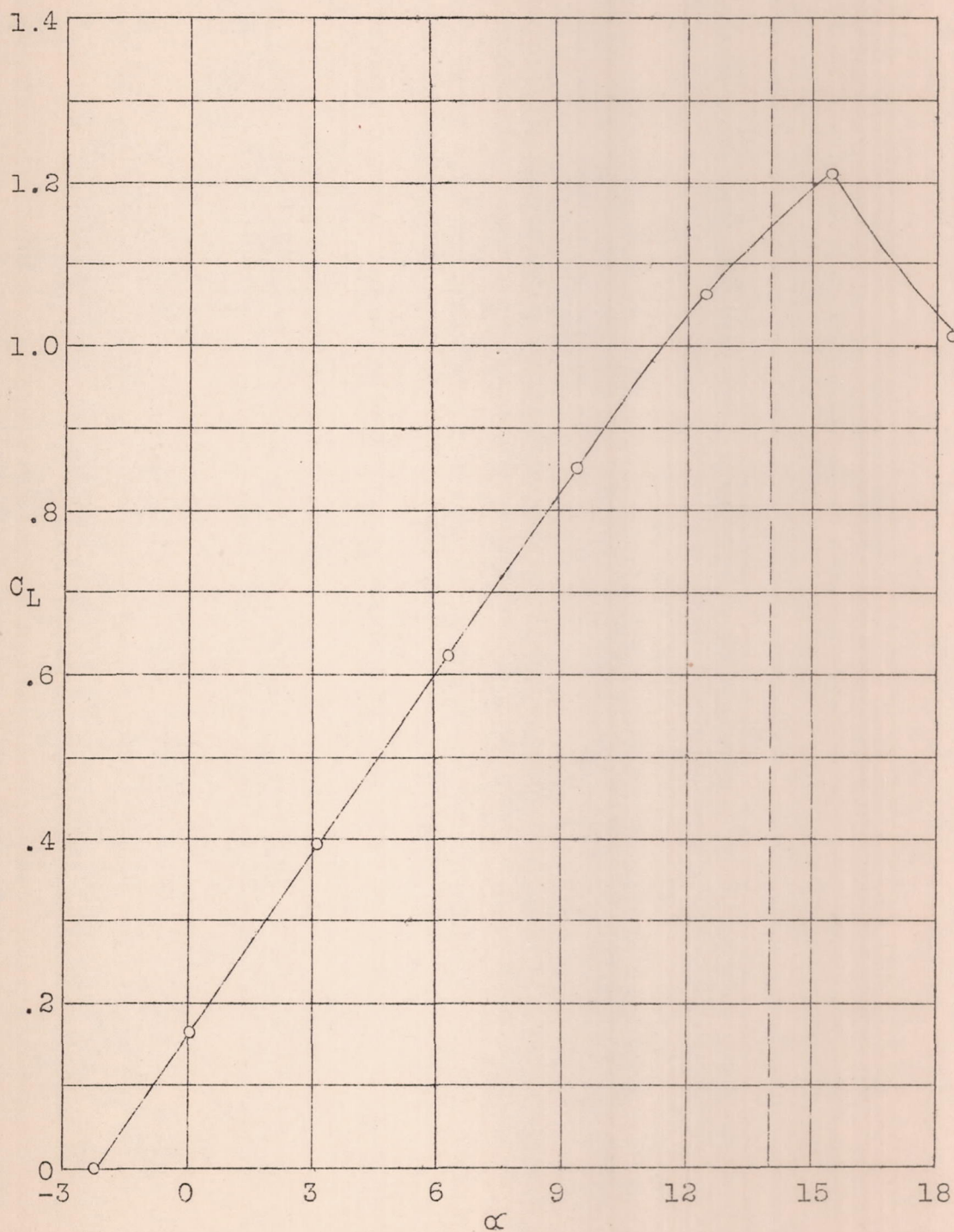


Fig. 7

SE-5A

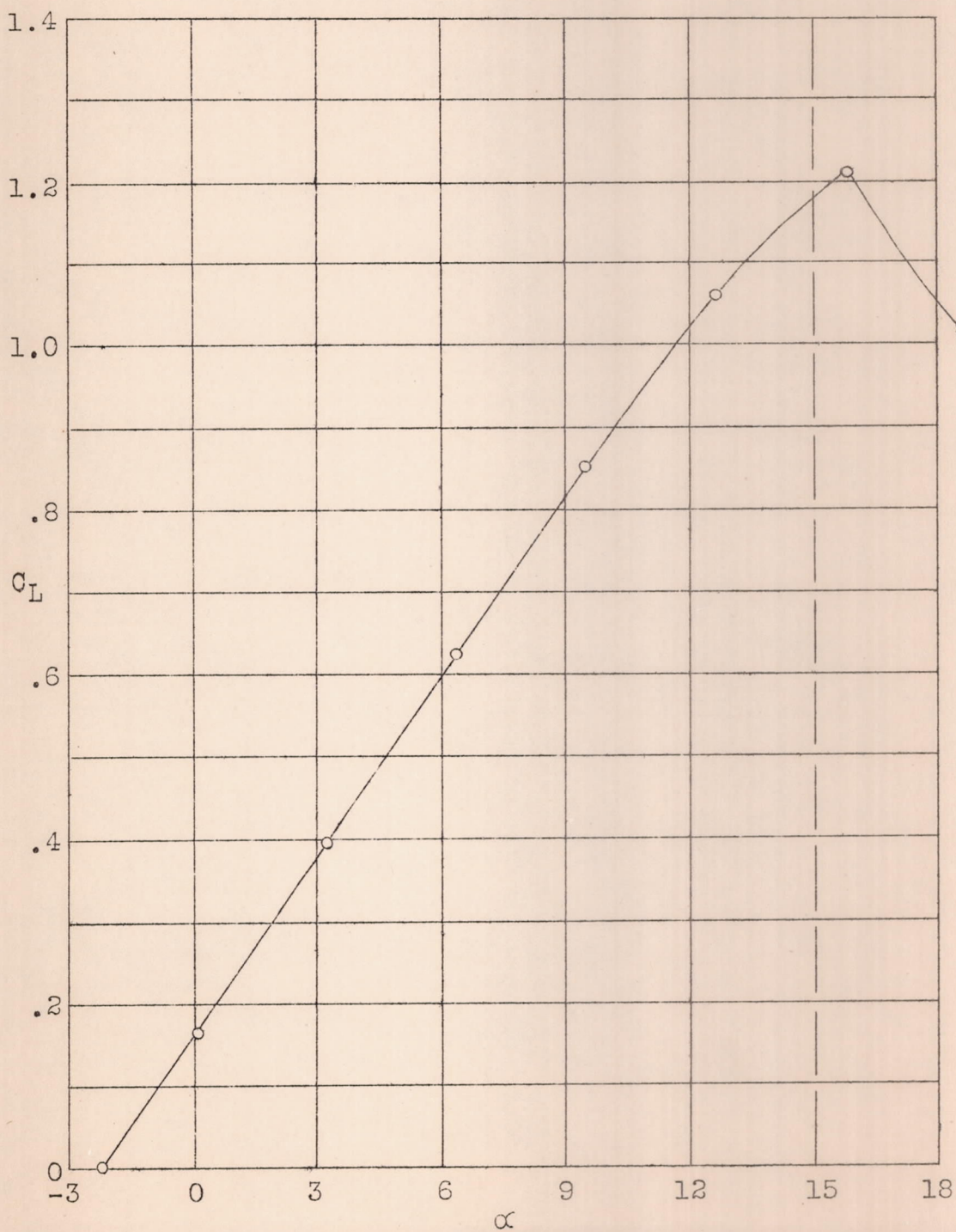


Fig. 8

Thomas Morse MB-3

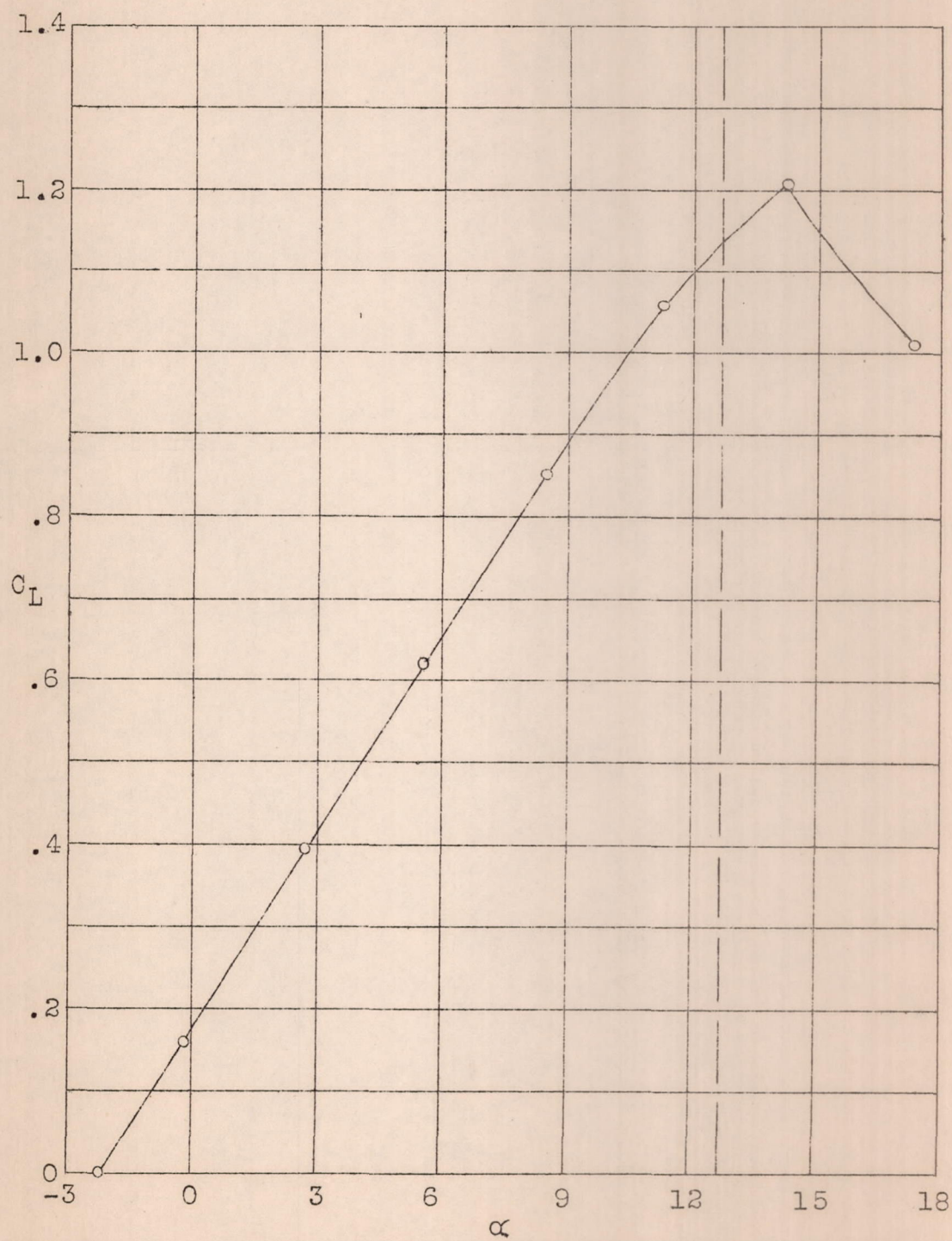


Fig. 9

Vought VE-7

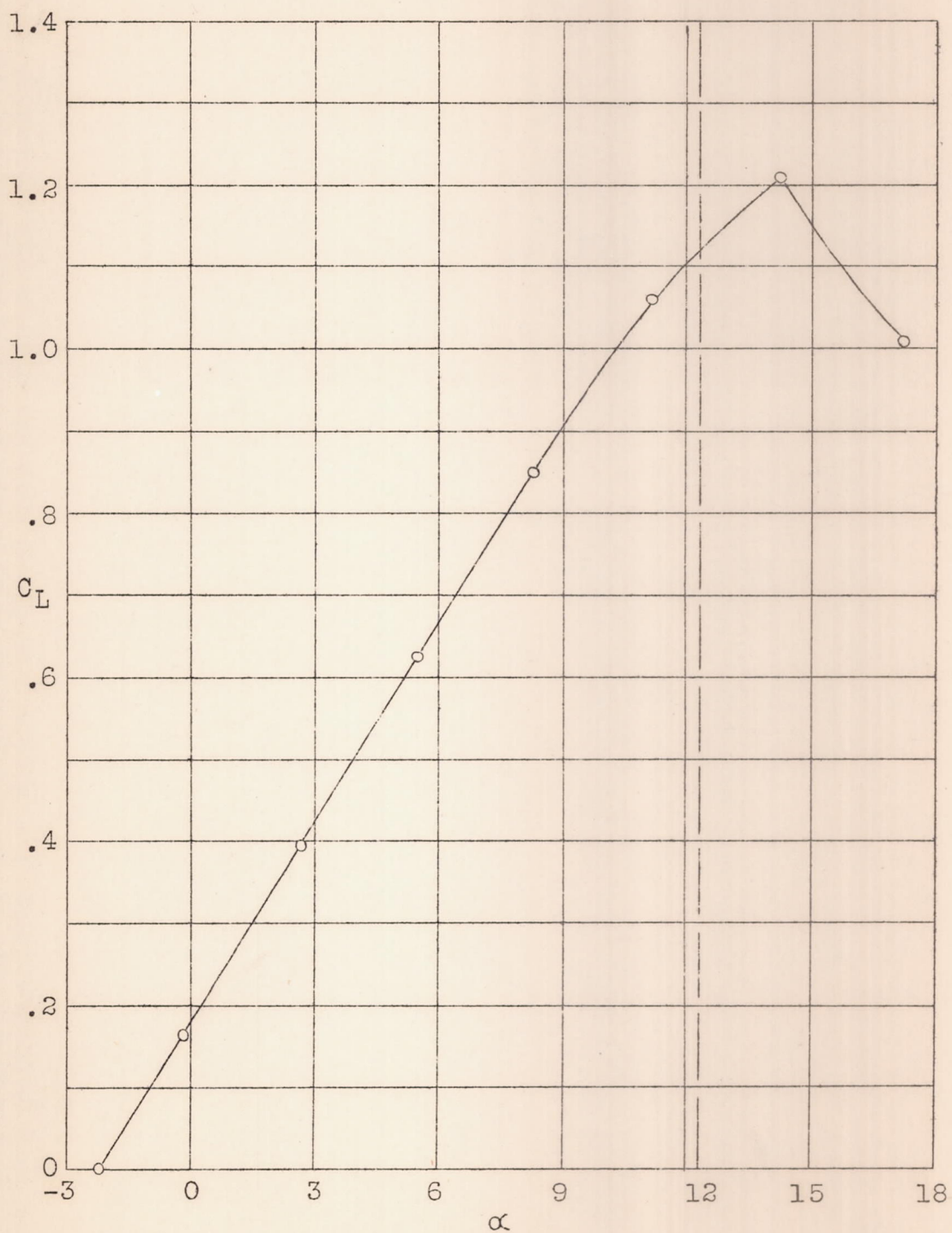


Fig. 10

DH-4B

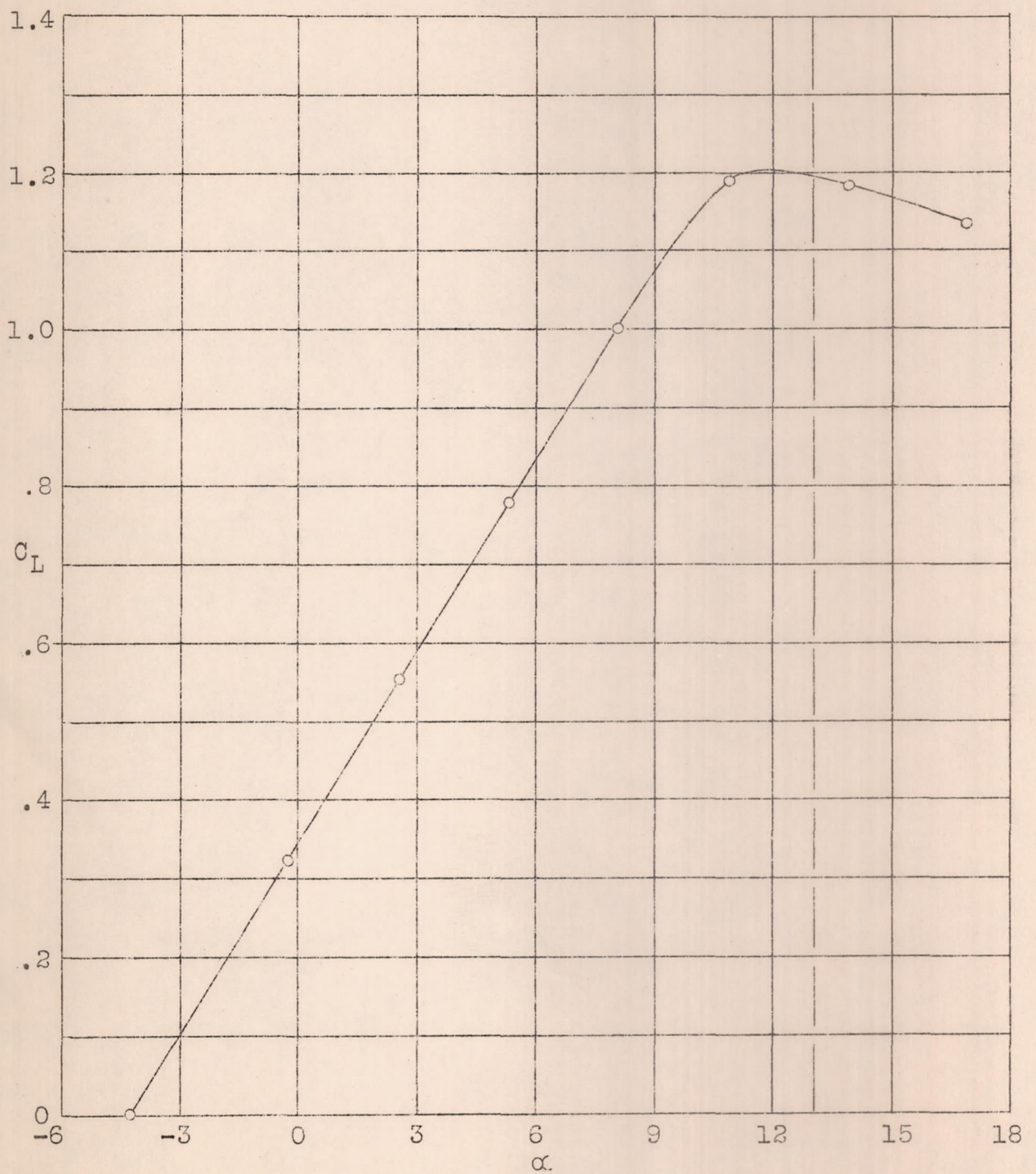


Fig. 11

Martin MB-II

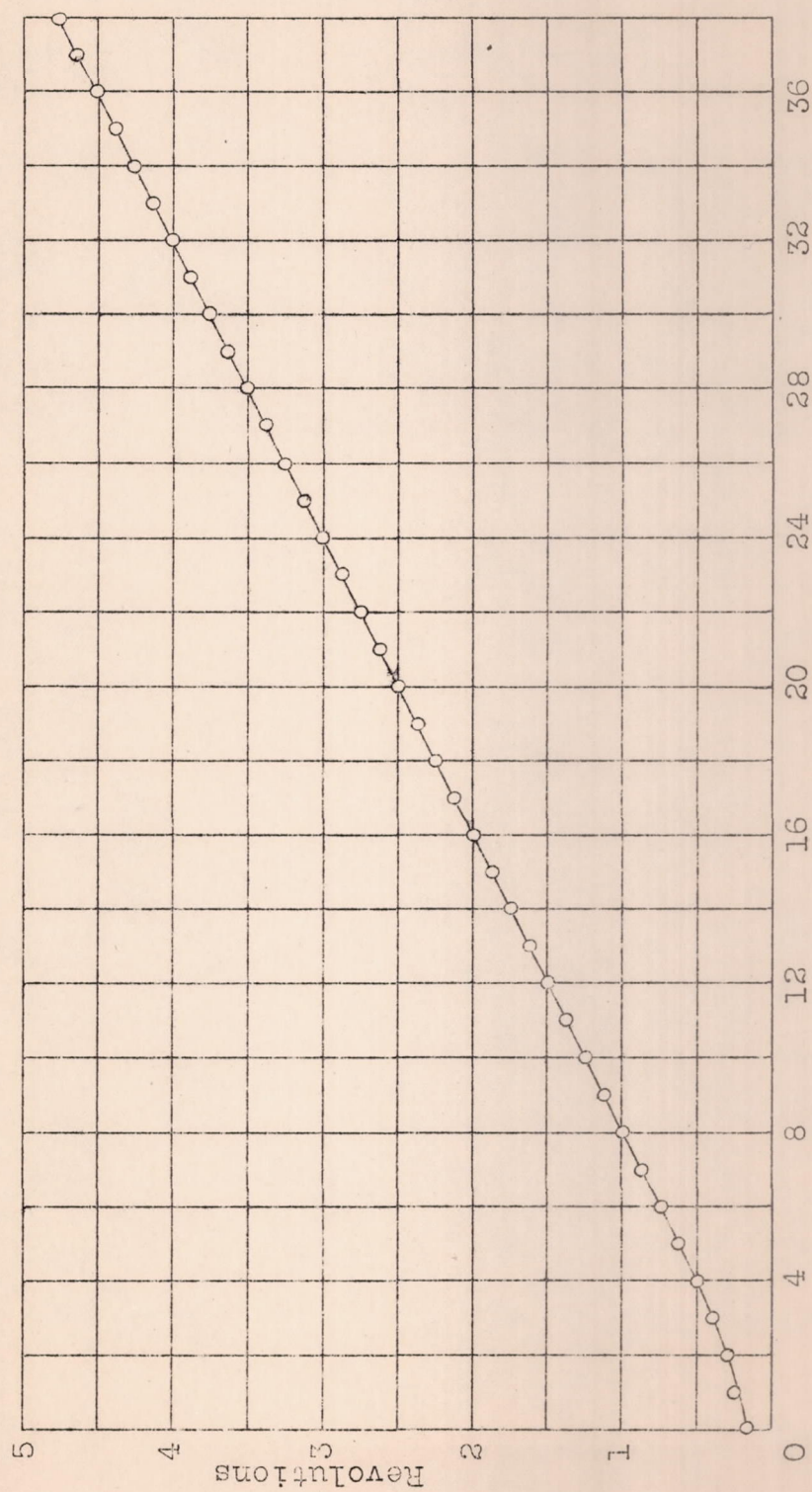


Fig. 13

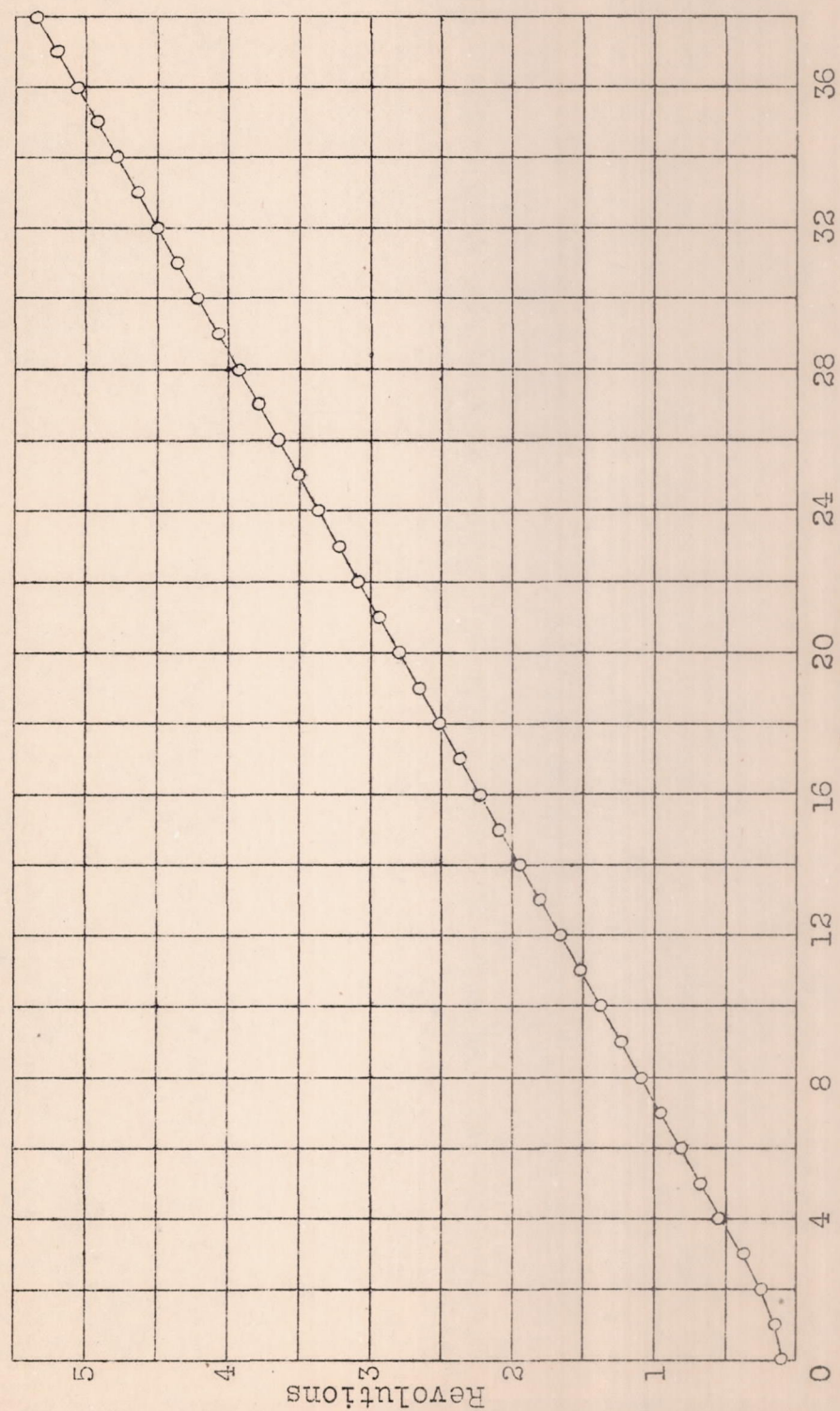


Fig. 13

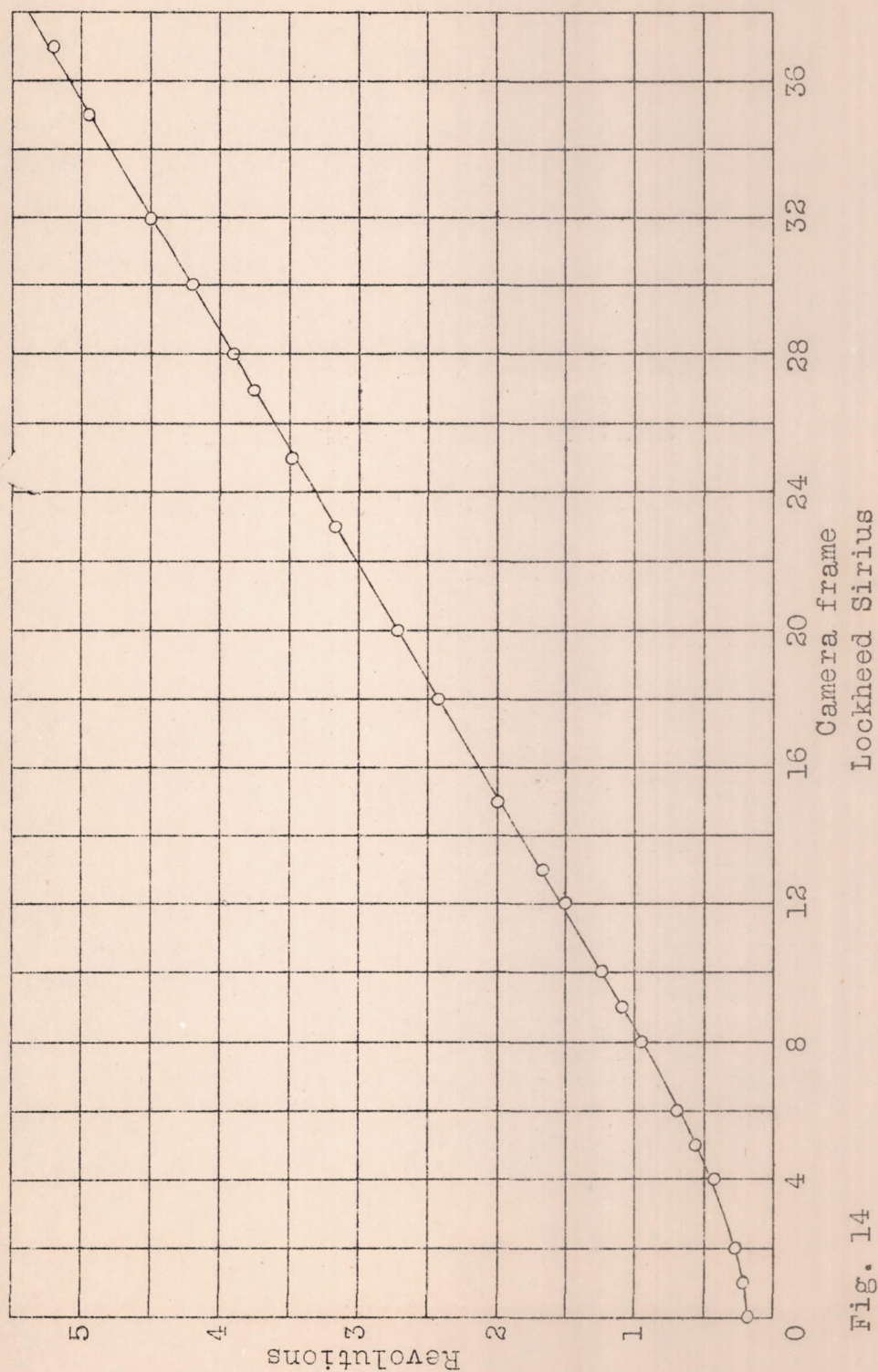


Fig. 14

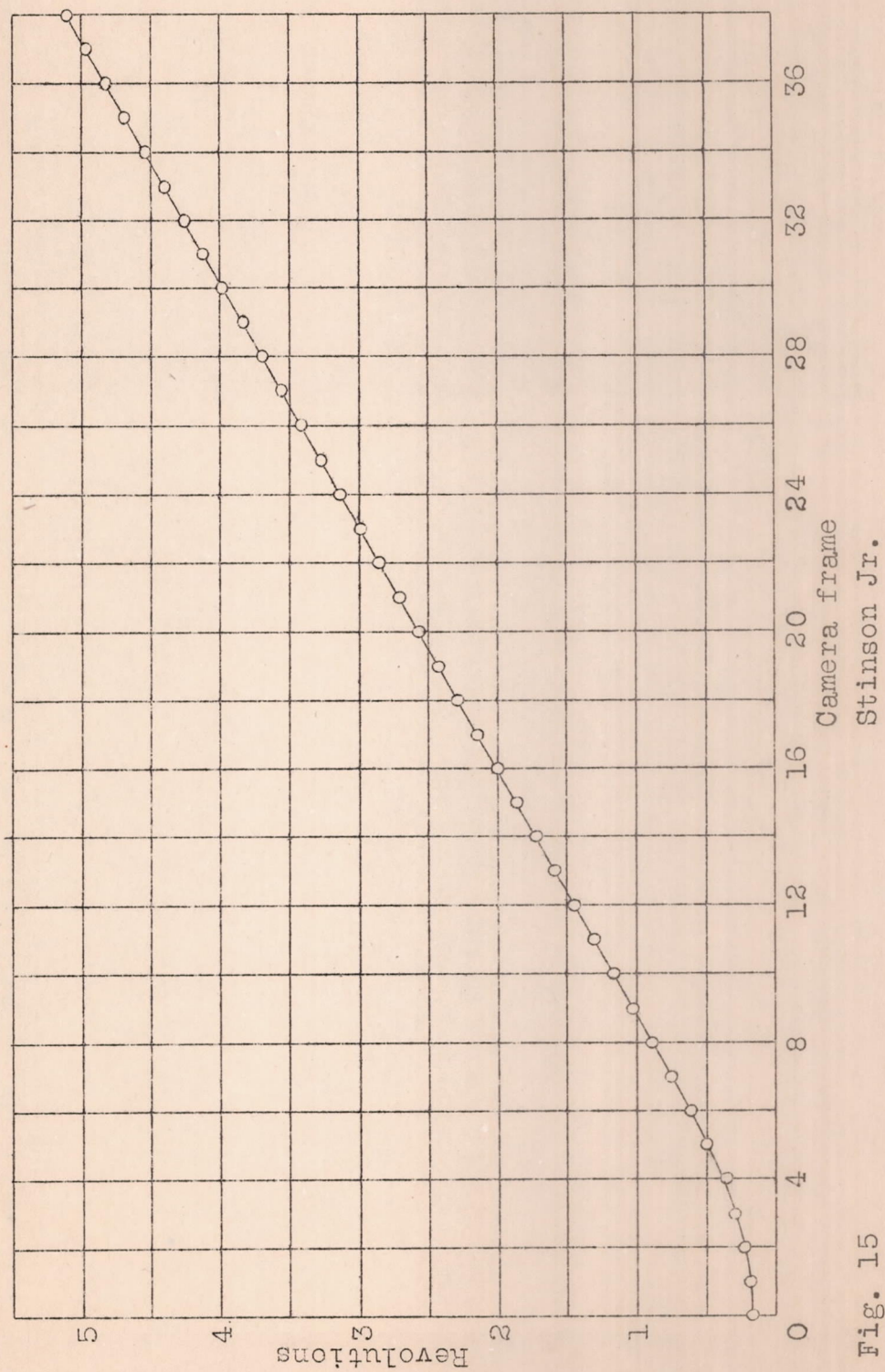


Fig. 15



Fig. 16

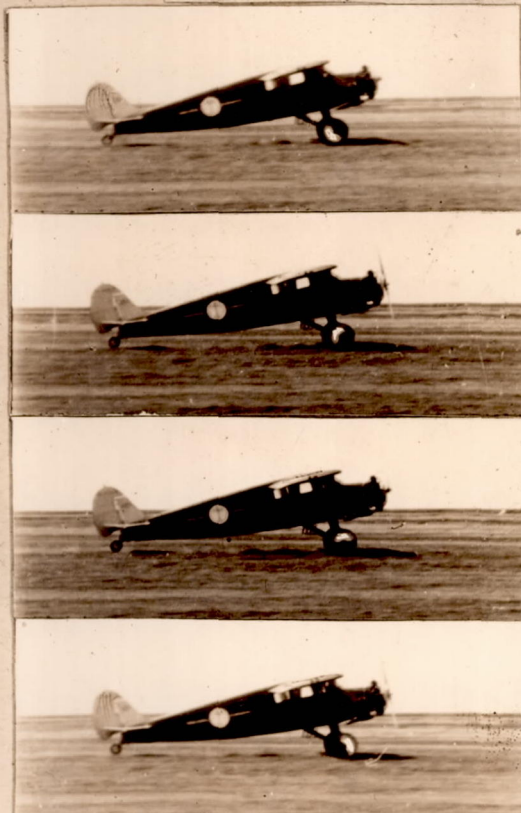


Fig. 18

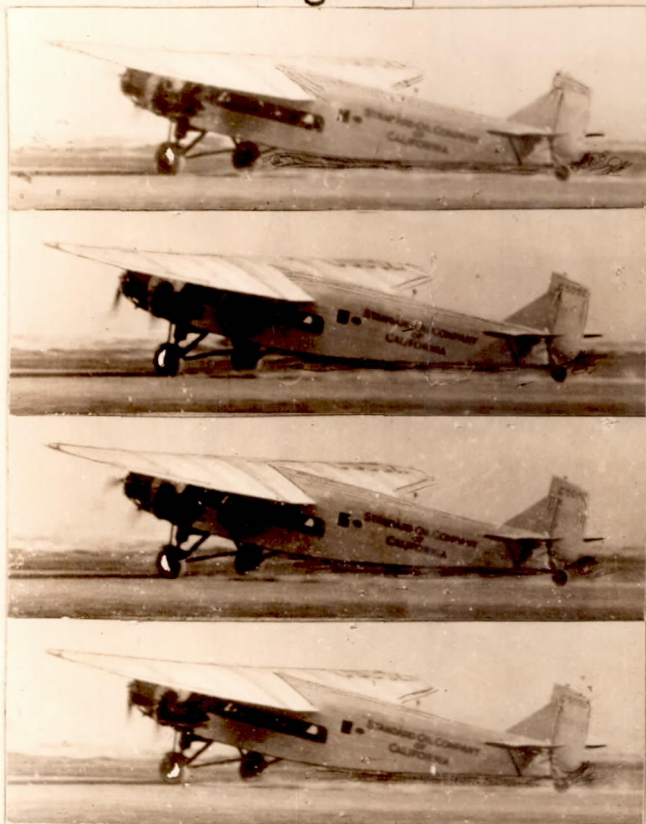


Fig. 17

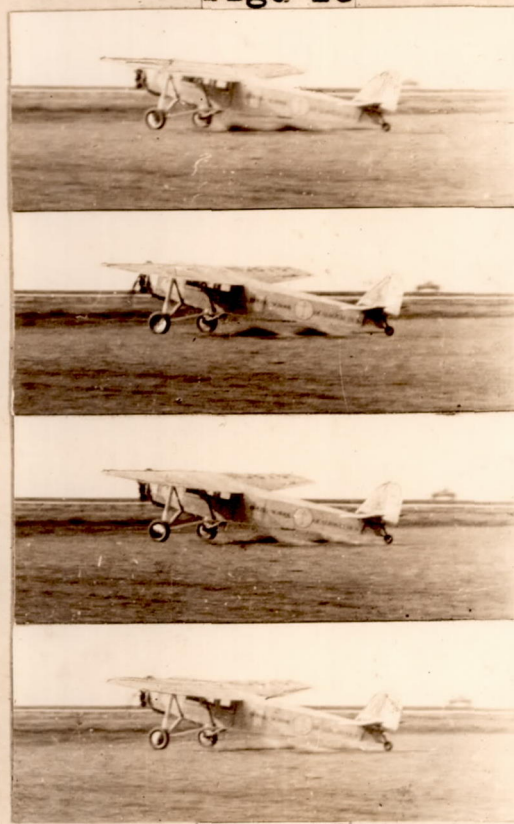


Fig. 19

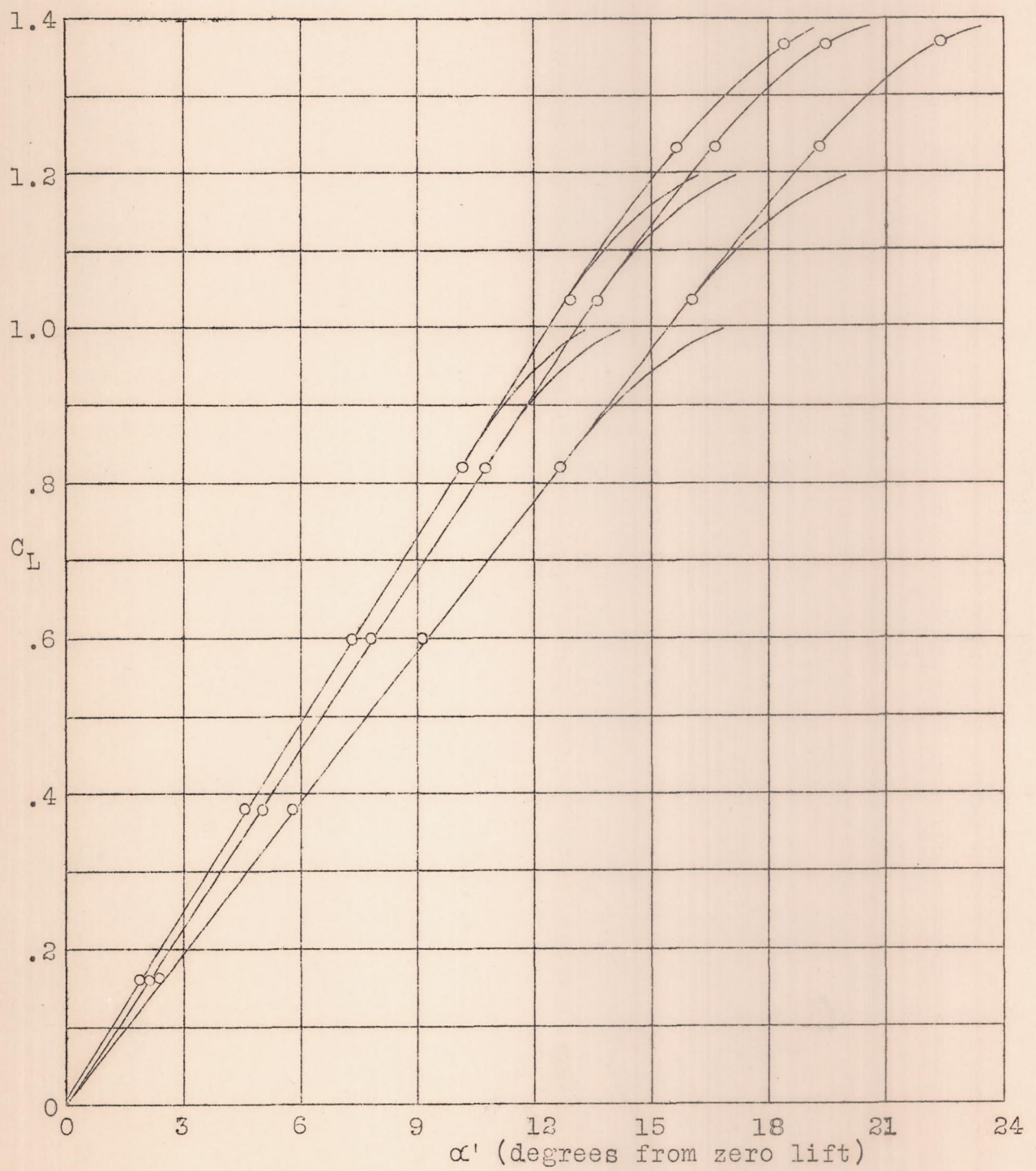


Fig. 20

Generalized lift curves.

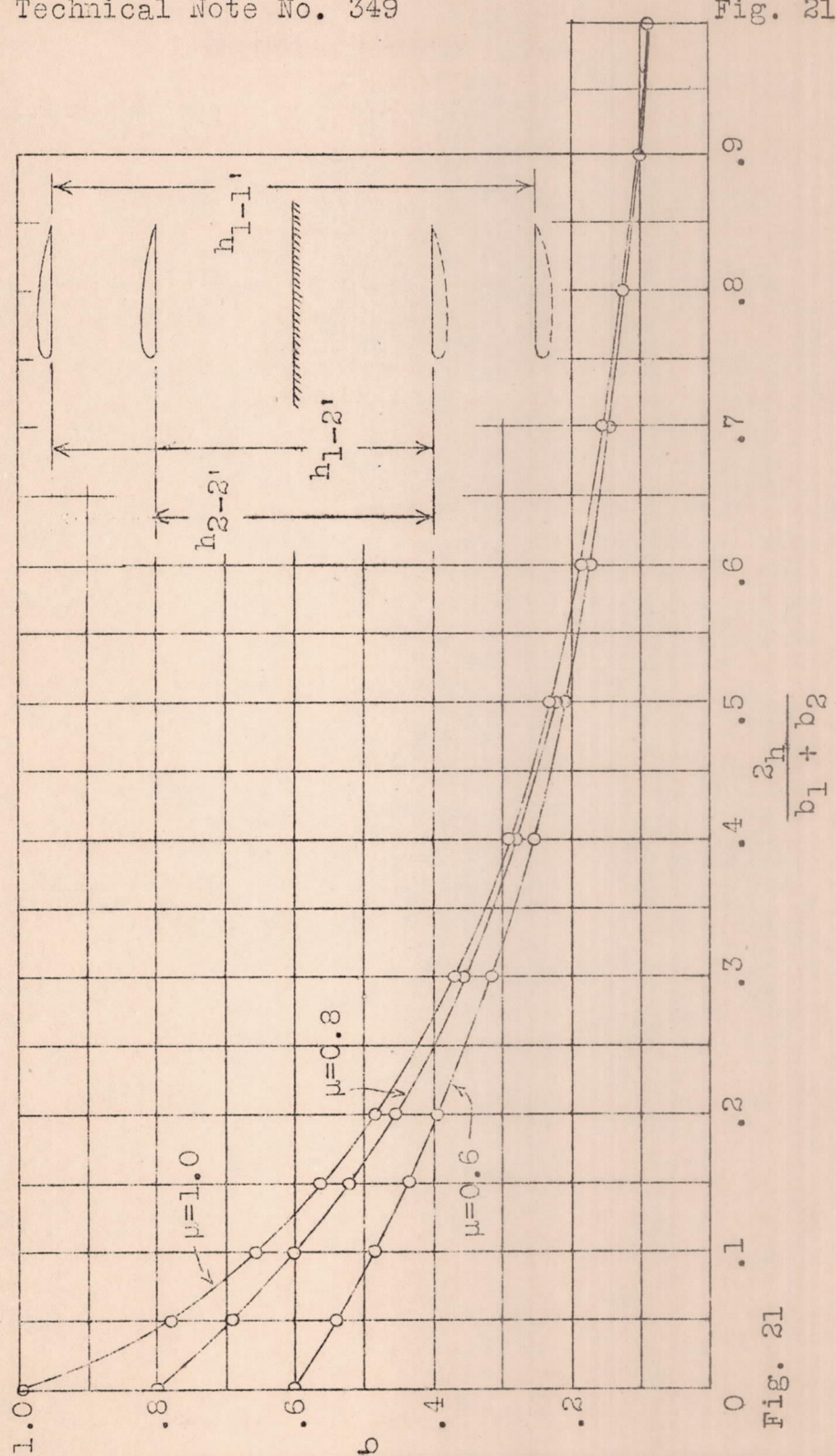


Fig. 21

Fig. 21